NOTES: Chapter 2

function: each element of the domain is paired with only one element in the range. (Use vertical line test to verify.)

f(x) → function notation…read as "f of x"

or "f at x"

EXAMPLE #1

Given: $g(x) = \frac{2}{3}x + \frac{1}{4}$

a. What is g(3)?

$g(3) = \frac{2}{\cancel{3}_1}(\cancel{3}^1) + \frac{1}{4}$

$= 2 + \frac{1}{4}$

$= \frac{8}{4} + \frac{1}{4} = \boxed{\frac{9}{4}}$

b. What is g(x+3)?

$g(x+3) = \frac{2}{3}(x+3) + \frac{1}{4}$

$= \frac{2}{3}x + 2 + \frac{1}{4}$

$= \boxed{\frac{2}{3}x + \frac{9}{4}}$

EXAMPLE #2: PIECEWISE FUNCTION

$$f(x)=\begin{cases} x^2-3 & \text{if } x<1 \\ 4x & \text{if } x\geq 1 \end{cases}$$

a. $f(3)= 4(3)$

$= \boxed{12}$

b. $f(-2)=(-2)^2-3$

$=4-3$

$=\boxed{1}$

Reminder: set notation

***SETS**

$\cup$ = union (all terms combined)

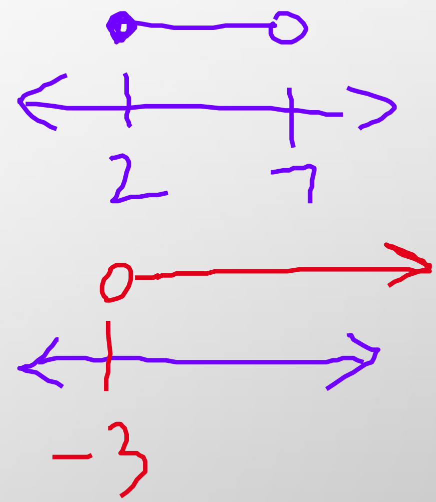
$\cap$ = intersection (common terms only)

***INTERVALS**

$[2, 7)$ $\quad 2 \leq x < 7$

$(-3, \infty)$ $\quad -3 < x < \infty$

or $\quad x > -3$



Reminder: Domain

Radical expressions: **(EVEN ROOTS ONLY!!)**

If given $\sqrt{x}$ → then solve $x \geq 0$

Fractional expressions:

If given $\frac{y}{x}$ → then solve $x \neq 0$

Example #3

State the domain. Show work and use interval notation for your final answer !

$\cancel{\{x \mid x \geq \frac{3}{2}\}}$

a. $f(x) = \sqrt{2x - 3}$

$2x - 3 \geq 0$

$2x \geq 3$

$\boxed{x \geq \frac{3}{2}} \longrightarrow \boxed{\left[\frac{3}{2}, \infty\right)}$

interval notation

b. $g(x) = \dfrac{5x}{2x - 3}$

$2x - 3 \neq 0$

$\boxed{x \neq \frac{3}{2}}$

$\boxed{\left(-\infty, \frac{3}{2}\right) \cup \left(\frac{3}{2}, \infty\right)}$

Notes: Transformations of Functions

Reflect over x-axis →

$$f(x) = -a(x-h)^2 + k$$

Stretch or compression →

← Shift up + or down -

← Shift left or right

Vertex = (h, k)

(x – 3) → shift *right* 3

(x + 3) → shift *left* 3

Note: move opposite of sign given inside parentheses

NOTES

Operations/Composition of Functions

$f(x) + g(x)$

$f(x) - g(x)$

multiply

$f(x) \bullet g(x)$ or $(f \bullet g)(x)$ compare to → $(f \circ g)(x)$ or $f(g(x))$

composition

$\frac{f(x)}{g(x)}$ or $\left(\frac{f}{g}\right)(x)$

$(g \circ f)(x)$ or $g(f(x))$

Examples: Given → $f(x) = 3x^2 - 4$ $g(x) = 4x + 5$

$(-\infty, \infty)$

or D: $x = \mathbb{R}$

Find:

1. $f(x) - g(x) = 3x^2 - 4 - (4x + 5)$

$= 3x^2 - 4 - 4x - 5 = \boxed{3x^2 - 4x - 9}$

2. $\left(\frac{f}{g}\right)(x) = \boxed{\frac{3x^2-4}{4x+5} \quad x \neq -\frac{5}{4}}$

or $\frac{f(x)}{g(x)}$

$4x + 5 \neq 0$

$4x \neq -5$

book $\left(-\infty, -\frac{5}{4}\right) \cup \left(-\frac{5}{4}, \infty\right)$

Examples: Given → $f(x) = 3x^2 - 4$ $\quad$ $g(x) = 4x + 5$

multiply

3. $f(x) \bullet g(x) = (3x^2-4)(4x+5)$

$= \boxed{12x^3 + 15x^2 - 16x - 20}$

+5

4x

4x+5

4. $(f \circ g)(x)$

or $f(g(x)) = 3(4x+5)^2 - 4$

Start

Substitute

75 −4

$= 3(16x^2 + 40x + 25) - 4$

$= \boxed{48x^2 + 120x + 71}$

Examples: Given → $f(x) = 3x^2 - 4$ $g(x) = 4x + 5$

5. $(g \circ f)(x)$

start

$= 4(3x^2 - 4) + 5$

$= 12x^2 - 16 + 5$

$= \boxed{12x^2 - 11}$

6. $(g \circ g)(x)$

start

$= 4(4x + 5) + 5$

$= 16x + 20 + 5$

$= \boxed{16x + 25}$

To solve for the inverse algebraically:

"swap" x and y values in the given equation, then solve the equation for y (rewrite in y- form.)

example: $f(x) = \frac{4}{13+x}$

rewrite: $y = \frac{4}{13+x}$

swap domain and range: $x = \frac{4}{13+y}$

now solve for y

swap domain and range: $x = \frac{4}{13+y}$

now solve for y

$$(13+y)x = \frac{4}{(13+y)}(13+y)$$

$$(13+y)x = 4$$

$$13+y = \frac{4}{x}$$

$$y = \frac{4}{x} - 13$$

$$\boxed{\begin{array}{l} f^{-1}(x) = \frac{4}{x} - 13 \\ \text{domain}: \ x \neq 0 \end{array}}$$

22. [-/1 Points] DETAILS SPRECALC7 2.8.055.

Find the inverse function of *f*.

$$f(x) = \frac{x}{x+4}$$

$f^{-1}(x) =$

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rewrite as $y = \frac{x}{x+4}$

then switch domain & range $y \to x$, $x \to y$

$$x = \frac{y}{y+4}$$

now put back in y= form

$$(y+4)\,x = \frac{y}{\cancel{y+4}}\,(y+4)$$

cancel denominator

distribute

$$xy + 4x = y$$

$$-xy \qquad -xy$$

gather like terms (all y values to one side)

$$4x = y - xy$$

factor y to isolate it on one side

$$4x = y(1 - x)$$

$$\frac{4x}{1-x} = y$$

now divide to get y by itself